

EXERCISE

Question 1:

A $100\ \Omega$ resistor is connected to a 220 V, 50 Hz ac supply.

- (a) What is the rms value of current in the circuit?
- (b) What is the net power consumed over a full cycle?

Solution 1:

Given:

$R = 100\ \text{ohms}$

$V = 220\text{V}$

Frequency(f) = 50Hz

(a) We know

$$I_{rms} = \frac{V_{rms}}{R}$$

Substituting the values $I_{rms} = \frac{220}{100} = 2.20\text{A}$

(b) Power = $V \cdot I$

Or Power = 220×2.2

Or Power = 484W

Question 2:

a) The peak voltage of an ac supply is 300 V. What is the rms voltage?

b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Solution 2:

a) Peak voltage of the ac supply, $V_0 = 300\text{V}$

We know $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.1\text{V}$

b) The rms value of current is given as $I = 10\text{A}$

Using above identity for current peak current is given as:

$I_0 = 1.414 \times I_{rms}$

Or $I_0 = 1.414 \times 10 = 14.14\ \text{A}$

Question 3:

A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Solution 3:

Given:

$$L = 44 \text{ mH} = 44 \times 10^{-3} \text{ H} = 44 \text{ mH}$$

$$\text{Frequency}(f) = 50 \text{ Hz}$$

$$\text{Angular frequency, } \omega = 2\pi f$$

$$\text{Inductive reactance, } X_L = \omega L = 2\pi fL$$

$$I_{rms} \text{ is given by } = \frac{V}{X_L}$$

Determining inductive reactance

$$X_L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3} \Omega$$

$$X_L = 13.82 \text{ ohms}$$

Therefore

$$I_{rms} = 220 / 13.82 = \frac{220}{13.82}$$

$$\text{Or } I_{rms} = 15.92 \text{ A}$$

Question 4:

A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Solution 4:

Given:

$$C = 60 \text{ microfarads}$$

$$V = 110 \text{ volts}$$

$$\text{Frequency}(f) = 60 \text{ Hz } I_{rms} = \frac{V}{X_c}$$

$$\text{Now, } X_c = \frac{1}{(2 \times 3.14 \times 60 \times 60 \times 10^{-6})}$$

$$X_c = 44.248 \text{ ohms}$$

$$\text{Hence, } I_{rms} = \frac{110}{44.248} = 2.488 \text{ A}$$

Question 5:

In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

Solution 5:

In the inductive circuit,

Rms value of current, $I=15.92\text{A}$

Rms value of voltage, $V=220\text{V}$

Hence the net power absorbed can be obtained by the relation,

$$P = VI \cos \phi$$

Where,

ϕ = Phase difference between V and I

For a pure inductive circuit, the phase difference alternating voltage and current is 90°

i.e., $\phi = 90^\circ$

Hence, $P=0$ i.e., the net power is zero.

In the capacitive circuit,

rms value of current, $I=2.49\text{A}$

rms value of voltage, $V=110\text{V}$

Hence the net power absorbed can be obtained as:

$$P = VI \cos \phi$$

For a pure capacitive, the phase difference between alternating voltage and current is 90°

i.e., $\phi = 90^\circ$

Hence, $P=0$, the net power is zero.

Question 6:

Obtain the resonant frequency ω_r of a series LCR circuit with $L = 2.0H$, $C = 32 \mu F$ and $R = 10 \Omega$. What is the Q-value of this circuit?

Solution 6:

Given:

$$L = 2 \text{ H}$$

$$C = 32 \mu F = 32 \times 10^{-6} \text{ F}$$

$$R = 10 \Omega$$

$$\text{Resonant frequency } \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{Substitution yields } = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = \frac{1}{8 \times 10^{-3}} = 125 \text{ rad / s}$$

Now Q-value = $\omega_r L / R$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$= \frac{1}{10} \sqrt{\frac{2}{32 \times 10^{-6}}}$$

$$= \frac{1}{10 \times 4 \times 10^{-3}} = 25$$

Hence, the Q-value is = 25

Question 7:

A charged $30 \mu F$ capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Solution 7:

Given:

$$C = 30 \mu F = 30 \times 10^{-6} \text{ F}$$

$$L = 27 \text{ mH } = 27 \times 10^{-3}$$

$$\text{Angular frequency of free oscillations } \omega_r = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{1}{9 \times 10^{-4}} = 1.11 \times 10^3 \text{ rad / s}$$

Hence, the angular frequency of free oscillations of the circuit is
 $= 1.1 \times 10^3 \text{ s}^{-1}$

Question 8:

Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC.. What is the total energy stored in the circuit initially? What is the total energy at later time?

Solution 8:

Capacitance of the capacitor, $C = 30 \mu\text{F} = 30 \times 10^{-6} \text{ F}$

Inductance of the inductor, $L = 27 \text{ mH} = 27 \times 10^{-3} \text{ H}$

Charge on the capacitor, $Q = 6 \text{ mC} = 6 \times 10^{-3} \text{ C}$

$$= E = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{(6 \times 10^{-3})^2}{30 \times 10^{-6}}$$

$$= \frac{6}{10} = 0.6 \text{ J}$$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

Question 9:

A series LCR circuit with $R = 20 \Omega$, $L = 1.5 \text{ H}$ and $C = 35 \mu\text{F}$ is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Solution 9:

Given:

$$R = 20 \Omega$$

$$L = 1.5 \text{ H}$$

$$C = 35 \mu\text{F} = 35 \times 10^{-6} \text{ F}$$

$$V = 200 \text{ volts}$$

Impedance of the circuit is given by the relation,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance ,

$$X_L = X_C$$

$$Z = R = 20 \Omega$$

$$\text{So } I = \frac{V}{Z} = \frac{200}{20} = 10A \text{ Thus,}$$

The average power transferred to the circuit in one complete cycle.

$$P = I^2 R$$

$$\text{Or } P = 10 \times 10 \times 20 = 2000 \text{ W}$$

Question 10:

A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200kHz). If its LC circuit has an effective inductance of $200 \mu\text{H}$, what must be the range of its variable capacitor?

[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radio wave.]

Solution 10:

The range of frequency (f) of a radio is 800KHZ to 1200KHZ

Effective inductance of circuit $L = 200 \mu\text{H} = 200 \times 10^{-6} \text{H}$

Capacitance of variable capacitor for f_1 is given as:

$$C_1 = \frac{1}{\omega_1^2 L} \text{ Where, } \omega_1 = \text{Angular frequency for capacitor for } f_1 = 2\pi f_1$$

$$2\pi \times 800 \times 10^3 \text{ rad / s}$$

$$\therefore C_1 = \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$$= 1.9809 \times 10^{-10} \text{F} = 198 \text{pf}$$

$$C_2 = \frac{1}{\omega_2^2 L}$$

$$\therefore C_2 = \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$$= 0.8804 \times 10^{-10} \text{F} = 88 \text{pF}$$

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

Question 11:

Figure shows a series LCR circuit connected to a variable frequency 230 V source. $L = 5.0 \text{ H}$, $C = 80 \mu\text{F}$, $R = 40 \Omega$.

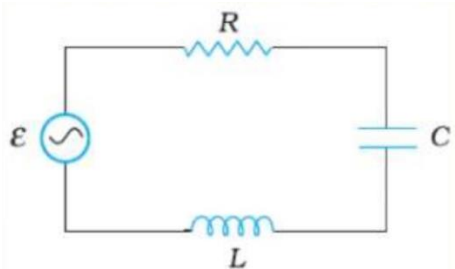


FIGURE 7.21

- Determine the source frequency which drives the circuit in resonance.
- Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
- Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Solution 11:

Given:

$$V = 230 \text{ V}$$

$$L = 5 \text{ H}$$

$$C = 80 \mu\text{F}$$

$$R = 40 \Omega$$

$$\begin{aligned} \text{(a) Source frequency at resonance} &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} \\ &= 50 \text{ rad / s} \end{aligned}$$

(b) At resonance,

Impedance, $Z = \text{Resistance, } R$

$$\text{So } Z = R = 40 \text{ ohms}$$

Now rms value of current,

$$I = \frac{V}{Z}$$

$$\text{Or } I = \frac{230}{40}$$

$$\text{Hence } I = 5.75 \text{ A}$$

Amplitude of this value of current = $1.414 \times I$

$$= 1.414 \times 5.75$$

$$= 8.13 \text{ A}$$

(c) Now taking into consideration the rms potential drops

Chapter 7
Alternating Current

Across Resistance,

$$V_R = IR = 5.75 \times 40$$

$$= 230 \text{ V}$$

Across Capacitance,

$$V_C = IX_C$$

$$= 1437.5 \text{ V}$$

Across Inductance,

$$V_L = IX_L$$

$$= 5.75 \times 50 \times 5$$

$$= 1437.5 \text{ V}$$

Across LC combination,

$$V_{LC} = I(X_L - X_C) \text{ At resonance ,}$$

$$X_L = X_C$$

$$\text{So, } V_{LC} = 0$$

Hence shown , that potential drop across LC combination is zero at resonanting frequency

Additional Exercise

Question 12:

An LC circuit contains a 20 mH inductor and a 50 μ F capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be $t = 0$.

- (a) What is the total energy stored initially? Is it conserved during LC oscillations?
- (b) What is the natural frequency of the circuit?
- (c) At what time is the energy stored (i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?
- (d) At what times is the total energy shared equally between the inductor and the capacitor?
- (e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Solution 12:

Inductance of the inductor, $L = 20 \text{ mH} = 20 \times 10^{-3} \text{ H}$

Capacitance of the capacitor, $C = 50 \mu\text{F} = 50 \times 10^{-6} \text{ F}$

Initial charge on the capacitor, $Q = 10 \text{ mC} = 10 \times 10^{-3} \text{ C}$

(a) Total energy stored initially in the circuit is given as:

$$E = \frac{1}{2} \frac{Q^2}{C}$$

$$= \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}} = 1 \text{ J}$$

Hence, the total energy stored in the LC circuit will be conserved because there is no resistor ($R=0$) connected in the circuit.

(b) Natural frequency of the circuit is given by the relation, $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}}$$

$$= \frac{10^3}{2\pi} = 159.24 \text{ Hz}$$

Natural angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = \frac{1}{\sqrt{10^{-6}}} = 10^3 \text{ rad/s}$$

Hence, the natural frequency of the circuit is 10^3 rad/s.

(c)(i) For time period $\left(T = \frac{1}{\nu} = \frac{1}{159.24} = 6.28 \text{ms} \right)$ total charge on the capacitor at time t ,

For energy stored is electrical, we can write $Q' = Q$.

Hence, it can be inferred that the energy stored in the capacitor is completely

electrical at time, $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \dots$

(ii) Magnetic energy is the maximum when electrical energy, Q' is equal to 0.

Hence, it can be inferred that the energy stored in the capacitor is completely

magnetic at time, $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \dots$ Where, $T = \frac{1}{f} = 6.3 \text{ms}$

(d) $Q^1 =$ Charge on the capacitor when total energy is equally shared between the capacitor and the inductor at time t .

When total energy is equally shared between the inductor and capacitor, the energy stored in the capacitor = $\frac{1}{2}$ (maximum energy)

$$\Rightarrow \frac{1}{2} \frac{(Q')^2}{C} = \frac{1}{2} \left(\frac{1}{2} \frac{Q^2}{C} \right) = \frac{1}{4} \frac{Q^2}{C}$$

$$Q' = \frac{Q}{\sqrt{2}}$$

$$\text{But } Q' = Q \cos \frac{2\pi}{T} t,$$

$$\frac{Q}{\sqrt{2}} = Q \cos \frac{2\pi}{T} t$$

$$\cos \frac{2\pi}{T} t = \frac{1}{\sqrt{2}} = \cos \left(2n+1 \right) \frac{\pi}{4} : \quad \text{where } n=0,1,2,3,\dots$$

$$t = (2n+1) \frac{T}{8}$$

Hence, total energy is equally shared between the inductor and the capacity at time,

$$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \dots$$

(e) If a resistor is inserted in the circuit, then total initial energy is dissipated as heat energy in the circuit. The resistance damps out the LC oscillation.

Question 13:

A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply.

- (a) What is the maximum current in the coil?
 (b) What is the time lag between the voltage maximum and the current maximum?

Solution 13:

Inductance of the inductor, $L = 0.50$ H

Resistance of the resistor, $R = 100$ Ω

Potential of the supply voltage, $V = 240$ V

Frequency of the supply, $\nu = 50$ Hz

(a) Peak voltage is given as:

$$V_0 = \sqrt{2V}$$

$$= \sqrt{2} \times 240 = 339.41V$$

Angular frequency of the supply, $\omega = 2\pi\nu$

$$= 2\pi \times 50 = 100\pi \text{ rad/s}$$

Maximum current in the circuit is given as:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$$

$$= \frac{339.41}{\sqrt{(100)^2 + (100\pi)^2 (0.50)^2}} = 1.82A$$

(b) Equation for voltage is given as: $V = V_0 \cos \omega t$

Equation for current is given as: $I = I_0 \cos (\omega t - \Phi)$

Where,

Φ = Phase difference between voltage and current At time, $t = 0$. $V = V_0$ (voltage is maximum)

For $\omega t - \Phi = 0$ i.e., at $t = \frac{\phi}{\omega}$, $I = I_0$ (current is maximum)

Hence, the time lag between maximum voltage and maximum current is $\frac{\phi}{\omega}$.

$$\tan \phi = \frac{\omega L}{R}$$

$$= \frac{2\pi \times 50 \times 0.5}{100} = 1.57$$

$$\phi = 57.5^\circ = \frac{57.5\pi}{180} \text{ rad}$$

$$\omega t = \frac{57.5}{180 \times 2\pi \times 50}$$

$$= 3.19 \times 10^{-3} \text{ s}$$

$$= 3.2 \text{ ms}$$

Now, phase angle Φ is given by the relation,

Hence, the time lag between maximum voltage and maximum current is 3.2 ms.

Question 14:

Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Solution 14:

Inductance of the inductor, $L = 0.5 \text{ H}$

Resistance of the resistor, $R = 100 \ \Omega$

Potential of the supply voltages, $V = 240 \text{ V}$

Frequency of the supply, $\nu = 10 \text{ kHz} = 10^4 \text{ Hz}$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 10^4 \text{ rad/s}$

Peak Voltage $V_0 = V\sqrt{2} = 110\sqrt{2} \text{ V}$

Maximum current, $I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$$= \frac{240\sqrt{2}}{\sqrt{(100)^2 + (2\pi \times 10^4)^2 \times (0.50)^2}} = 1.1 \times 10^{-2} \text{ A}$$

(b) For phase difference Φ , we have the relation:

$$\begin{aligned} \tan \phi &= \frac{\omega L}{R} \\ &= \frac{2\pi \times 10^4 \times 0.5}{100} = 100\pi \end{aligned}$$

$$\phi = 89.82^\circ = \frac{89.82\pi}{180} \text{ rad}$$

$$\omega t = \frac{89.82\pi}{180}$$

$$t = \frac{89.82\pi}{180 \times 2\pi \times 10^4} = 25 \mu\text{s}$$

It can be observed that I_0 is very small in this case. Hence, at high frequencies, the inductor amounts to an open circuit.

In a dc circuit, after a steady state is achieved, $\omega = 0$. Hence, inductor L behaves like a pure conducting object.

Question 15:

A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.

- (a) What is the maximum current in the circuit?
 (b) What is the time lag between the current maximum and the voltage maximum?

Solution 15:

Capacitance of the capacitor, $C = 100 \mu\text{F} = 100 \times 10^{-6} \text{ F}$

Resistance of the resistor, $R = 40 \Omega$

Supply voltage, $V = 110 \text{ V}$

Frequency of oscillations, $\nu = 60 \text{ Hz}$

(b) Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60 \text{ rad/s}$

For a RC circuit, we have the relation for impedance as: $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Peak Voltage $V_0 = V\sqrt{2} = 110\sqrt{2} \text{ V}$

Maximum current is given as: $I_0 = \frac{V_0}{Z}$

$$\begin{aligned}
 &= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \\
 &= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}} \\
 &= \frac{110\sqrt{2}}{\sqrt{1600 + \frac{10^2}{(120\pi)^2}}} = 3.24 \text{ A}
 \end{aligned}$$

(b) In a capacitor circuit, the voltage lags behind the current by a phase angle of Φ . This angle is given by the relation:

$$\begin{aligned} \therefore \tan \phi &= \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR} \\ &= \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6635 \\ \phi &= \tan^{-1}(0.6635) = 33.56^\circ \\ &= \frac{33.56\pi}{180} \text{ rad} \\ \therefore \text{Time lag} &= \frac{\phi}{\omega} \\ &= \frac{33.56\pi}{180 \times 120\pi} = 1.55 \times 10^{-3} \text{ s} = 1.55 \text{ ms} \end{aligned}$$

Hence, the time lag between maximum current and maximum voltage is 1.55 ms.

Question 16:

Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Solution 16:

Capacitance of the capacitor, $C = 100 \mu\text{F} = 100 \times 10^{-6} \text{ F}$

Resistance of the resistor, $R = 40 \Omega$

Supply voltage, $V = 110 \text{ V}$

Frequency of the supply, $\nu = 12 \text{ kHz} = 12 \times 10^3 \text{ Hz}$

Angular Frequency, $\omega = 2\pi\nu = 2 \times \pi \times 12 \times 10^3 \text{ rad/s}$

$= 24\pi \times 10^3 \text{ rad/s}$

Peak Voltage $V_0 = V\sqrt{2} = 110\sqrt{2} \text{ V}$

Maximum current, $I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

$$= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(24\pi \times 10^3 \times 100 \times 10^{-6})^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{1600 + \left(\frac{10}{24\pi}\right)^2}} = 3.9A$$

For an RC circuit, the voltage lags behind the current by a phase angle of Φ given as:

$$\tan \phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$= \frac{1}{24\pi \times 10^3 \times 100 \times 10^{-6} \times 40}$$

$$\tan \phi = \frac{1}{96\pi}$$

$$\therefore \phi = 0.2^\circ$$

$$\text{Time lag} = \frac{\phi}{\omega}$$

$$= \frac{0.2\pi}{180 \times 24\pi \times 10^3} = 1.55 \times 10^{-3} s = 0.04 \mu s$$

Hence, Φ tends to become zero at high frequencies. At a high frequency, capacitor C acts as a conductor.

In a dc circuit, after the steady state is achieved, $\omega = 0$. Hence, capacitor C amounts to an open circuit.

Question 17:

Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L, C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

Solution 17:

An inductor (L), a capacitor (C), and a resistor (R) is connected in parallel with each other in a circuit where,

$$L = 5.0 \text{ H}$$

$$C = 80 \mu\text{F} = 80 \times 10^{-6} \text{ F}$$

$$R = 40 \Omega$$

Potential of the voltage source, $V = 230 \text{ V}$

Impedance (Z) of the given parallel LCR circuit is given as:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Where, $\omega = \text{Angular frequency}$

At resonance $\frac{1}{\omega L} - \omega C = 0$

$$\begin{aligned} \therefore \omega &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{ rad/s} \end{aligned}$$

Hence, the magnitude of Z is the maximum at 50 rad/s. As a result, the total current is minimum. rms current flowing through inductor L is given as:

$$\begin{aligned} I_L &= \frac{V}{\omega L} \\ &= \frac{230}{50 \times 5} = 0.92 \text{ A} \end{aligned}$$

rms current flowing through capacitor C is given as:

$$\begin{aligned} I_c &= \frac{V}{\frac{1}{\omega C}} = \omega CV \\ &= 50 \times 80 \times 10^{-6} \times 230 = 0.92 \text{ A} \end{aligned}$$

rms current flowing through resistor R is given as:

$$\begin{aligned} I_R &= \frac{V}{R} \\ &= \frac{230}{40} = 5.75 \text{ A} \end{aligned}$$

Question 18:

A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

- Obtain the current amplitude and rms values.
- Obtain the rms values of potential drops across each element.
- What is the average power transferred to the inductor?
- What is the average power transferred to the capacitor?
- What is the total average power absorbed by the circuit? [‘Average’ implies ‘averaged over one cycle’.]

Solution 18:

Inductance, $L = 80 \text{ mH} = 80 \times 10^{-3} \text{ H}$

Capacitance, $C = 60 \mu\text{F} = 60 \times 10^{-6} \text{ F}$

Supply voltage, $V = 230 \text{ V}$

Frequency, $\nu = 50 \text{ Hz}$

Angular frequency, $\omega = 2\pi\nu = 100\pi \text{ rad/s}$

Peak voltage, $V_0 = V\sqrt{2} = 230\sqrt{2} \text{ V}$

(a) Maximum current is given as: Hence, rms value of current,

$$\begin{aligned}
 I_0 &= \frac{V_0}{\left(\omega L - \frac{1}{\omega C}\right)} \\
 &= \frac{230\sqrt{2}}{\left(100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)} \\
 &= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = -11.63 \text{ A}
 \end{aligned}$$

The negative sign appears because $\omega L < \frac{1}{\omega C}$

Amplitude of maximum current, $|I_0| = 11.63 \text{ A}$

$$I = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}} = -8.22 \text{ A}$$

(b) Potential difference across the inductor, $V_L = I \times \omega L$

$$\begin{aligned}
 &= 8.22 \times 100\pi \times 80 \times 10^{-3} \\
 &= 206.61 \text{ V}
 \end{aligned}$$

Potential difference across the capacitor,

$$\begin{aligned}
 V_C &= I \times \frac{1}{\omega C} \\
 &= 8.22 \times \frac{1}{100\pi \times 60 \times 10^{-6}} = 436.3 \text{ V}
 \end{aligned}$$

(c) Average power consumed by the inductor is zero as actual voltage leads the current by $\frac{\pi}{2}$

(d) Average power consumed by the capacitor is zero as voltage lags current by $\frac{\pi}{2}$

(e) The total power absorbed (averaged over one cycle) is zero.

Question 19:

Suppose the circuit in Exercise 7.18 has a resistance of 15Ω . Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Solution 19:

Average power transferred to the resistor = 788.44 W

Average power transferred to the capacitor = 0 W

Total power absorbed by the circuit = 788.44 W

Inductance of inductor, $L = 80 \text{ mH} = 80 \times 10^{-3} \text{ H}$

Capacitance of capacitor, $C = 60 \text{ }\mu\text{F} = 60 \times 10^{-6} \text{ F}$

Resistance of resistor, $R = 15 \text{ }\Omega$

Potential of voltage supply, $V = 230 \text{ V}$

Frequency of signal, $\nu = 50 \text{ Hz}$

Angular frequency of signal, $\omega = 2\pi\nu = 2\pi \times (50) = 100\pi \text{ rad/s}$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(15)^2 + \left[100\pi(80 \times 10^{-3}) - \frac{1}{(100\pi \times 60 \times 10^{-6})}\right]^2}$$

$$= \sqrt{(15)^2 + (25.12 - 53.08)^2} = 31.728 \Omega$$

$$I = \frac{V}{Z} = \frac{230}{31.728} = 7.25 \text{ A}$$

The elements are connected in series to each other. Hence, impedance of the circuit is given as
Current flowing in the circuit,

Average power transferred to resistance is given as: $P_R = I^2 R$

$$= (7.25)^2 \times 15 = 788.44 \text{ W}$$

Average power transferred to capacitor, $P_C =$ Average power transferred to inductor, $P_L = 0$

Total power absorbed by the circuit:

$$= P_R + P_C + P_L$$

$$= 788.44 + 0 + 0 = 788.44 \text{ W}$$

Hence, the total power absorbed by the circuit is 788.44 W.

Question 20:

A series LCR circuit with $L = 0.12 \text{ H}$, $C = 480 \text{ nF}$, $R = 23 \text{ }\Omega$ is connected to a 230 V variable frequency supply.

- What is the source frequency for which current amplitude is maximum? Obtain this maximum value.
- What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.
- For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the Q-factor of the given circuit?

Solution 20:

Inductance, $L = 0.12 \text{ H}$

Capacitance, $C = 480 \text{ nF} = 480 \times 10^{-9} \text{ F}$ Resistance, $R = 23 \Omega$

Supply voltage, $V = 230 \text{ V}$ Peak voltage is given as:

$$V_0 = \sqrt{2} \times 230 = 325.22 \text{ V}$$

(a) Current flowing in the circuit is given by the relation,
$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Where,

$$I_0 = \text{maximum at resonance At resonance, we have } \omega_R L - \frac{1}{\omega_R C} = 0$$

Where,

$\omega_R =$ Resonance angular frequency

$$\begin{aligned} \omega_R &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = 4166.67 \text{ rad / s} \end{aligned}$$

$$\text{Resonant Frequency, } \nu_R = \frac{\omega_R}{2\pi} = \frac{4166.67}{2 \times 3.14} = 663.48 \text{ Hz}$$

$$\text{And, maximum current } (I_0)_{\text{Max}} = \frac{V_0}{R} = \frac{325.22}{23} = 14.14 \text{ A}$$

(b) Maximum average power absorbed by the circuit is given as:

$$\begin{aligned} (P_V)_{\text{Max}} &= \frac{1}{2} (I_0)_{\text{max}}^2 R \\ &= \frac{1}{2} \times (14.14)^2 \times 23 = 2299.3 \text{ W} \end{aligned}$$

Hence, resonant frequency, (ν_R) is 663.48 Hz

(c) The power transferred to the circuit is half the power at resonant frequency.

Frequencies at which power transferred is half, $= \omega_R \pm \Delta\omega$

$$= 2\pi(\nu_R \pm \Delta\nu)$$

Where,

$$\begin{aligned}\Delta\omega &= \frac{R}{2L} \\ &= \frac{23}{2 \times 0.12} = 95.83 \text{ rad/s}\end{aligned}$$

$$\text{Hence, change in frequency } \Delta\nu = \frac{1}{2\pi} \Delta\omega = \frac{95.83}{2\pi} = 15.26 \text{ Hz}$$

$$\nu_R + \Delta\nu = 663.48 + 15.26 = 678.74 \text{ Hz}$$

$$\text{and } \nu_R - \Delta\nu = 663.48 - 15.26 = 648.22 \text{ Hz}$$

Hence, at 648.22 Hz and 678.74 Hz frequencies, the power transferred is half.

At these frequencies, current amplitude can be given as:

$$\begin{aligned}I' &= \frac{1}{\sqrt{2}} \times (I_0)_{\text{Max}} \\ &= \frac{14.14}{\sqrt{2}} = 10 \text{ A}\end{aligned}$$

(d) Q-factor of the given circuit can be obtained using the relation,

$$Q = \frac{\omega_R L}{R} = \frac{4166.67 \times 0.12}{23} = 21.74$$

Hence, the Q-factor of the given circuit is 21.74.

Question 21:

Obtain the resonant frequency and Q-factor of a series LCR circuit with $L = 3.0 \text{ H}$, $C = 27 \mu\text{F}$, and $R = 7.4 \Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its ‘full width at half maximum’ by a factor of 2. Suggest a suitable way.

Solution 21:

Inductance, $L = 3.0 \text{ H}$

Capacitance, $C = 27 \mu\text{F} = 27 \times 10^{-6} \text{ F}$ Resistance, $R = 7.4 \Omega$

At resonance, angular frequency of the source for the given LCR series circuit is given as:

$$\begin{aligned}\omega_r &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = \frac{10^3}{9} \\ &= 111.11 \text{ rad s}^{-1}\end{aligned}$$

Q-factor of the series:

$$\begin{aligned}Q &= \frac{\omega_r L}{R} \\ &= \frac{111.11 \times 3}{7.4} = 45.0446\end{aligned}$$

To improve the sharpness of the resonance by reducing its 'full width at half maximum' by a factor of 2 without changing ω_r , we need to reduce R to half i.e., Resistance =

$$\frac{R}{2} = \frac{7.4}{2} = 3.7\Omega$$

Question 22:

Answer the following questions:

- (a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?
- (b) A capacitor is used in the primary circuit of an induction coil.
- (c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series.

Show that the dc signal will appear across C and the ac signal across L.

- (d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.
- (e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Solution 22:

- (a) Yes, the statement is not true for rms voltage

It is true that in any ac circuit, the applied voltage is equal to the average sum of the instantaneous voltages across the series elements of the circuit. However, this is not true for rms voltage because voltages across different elements may not be in phase.

- (b) High induced voltage is used to charge the capacitor.

A capacitor is used in the primary circuit of an induction coil. This is because when the circuit is broken, a high induced voltage is used to charge the capacitor to avoid sparks.

- (c) The dc signal will appear across capacitor C because for dc signals, the impedance of an inductor (L) is negligible while the impedance of a capacitor (C) is very high (almost infinite). Hence, a dc signal appears across C. For an ac signal of high frequency, the impedance of L is high and that of C is very low.

Hence, an ac signal of high frequency appears across L.

- (d) If an iron core is inserted in the choke coil (which is in series with a lamp connected to the ac line), then the lamp will glow dimly. This is because the choke coil and the iron core increase the impedance of the circuit.

- (e) A choke coil is needed in the use of fluorescent tubes with ac mains because it reduces the voltage across the tube without wasting much power. An ordinary resistor cannot be used instead

of a choke coil for this purpose because it wastes power in the form of heat.

Question 23:

A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

Solution 23:

Input voltage, $V_1 = 2300$

Number of turns in primary coil, $n_1 = 4000$

Output voltage, $V_2 = 230$ V

Number of turns in secondary coil = n_2

Voltage is related to the number of turns as: $\frac{V_1}{V_2} = \frac{n_1}{n_2}$

$$\frac{2300}{230} = \frac{4000}{n_2}$$

$$n_2 = \frac{4000 \times 230}{2300} = 400$$

Hence, there are 400 turns in the second winding.

Question 24:

At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is $100 \text{ m}^3 \text{ s}^{-1}$. If the turbine generator efficiency is 60%, estimate the electric power available from the plant ($g = 9.8 \text{ m s}^{-2}$).

Solution 24:

Height of water pressure head, $h = 300$ m

Volume of water flow per second, $V = 100 \text{ m}^3/\text{s}$

Efficiency of turbine generator, $\eta = 60\% = 0.6$

Acceleration due to gravity, $g = 9.8 \text{ m/s}^2$

Density of water, $\rho = 10^3 \text{ kg/m}^3$

Electric power available from the plant = $\eta \times h\rho gV$

$$= 0.6 \times 300 \times 10^3 \times 9.8 \times 100$$

$$= 176.4 \times 10^6 \text{ W}$$

$$= 176.4 \text{ MW}$$

Question 25:

A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 400-220 V step-down transformer at a sub-station in the town.

- (a) Estimate the line power loss in the form of heat.
- (b) How much power must the plant supply, assuming there is negligible power loss due to leakage?
- (c) Characterise the step up transformer at the plant.

Solution 25:

Total electric power required, $P = 800 \text{ kW} = 800 \times 10^3 \text{ W}$

Supply voltage, $V = 220 \text{ V}$

Voltage at which electric plant is generating power, $V' = 440 \text{ V}$

Distance between the town and power generating station, $d = 15 \text{ km}$

Resistance of the two wire lines carrying power = 0.5 Ω/km

Total resistance of the wires, $R = (15 + 15)0.5 = 15 \Omega$

A step-down transformer of rating 4000 – 220 V is used in the sub-station. Input voltage, $V_1 = 4000 \text{ V}$

Output voltage, $V_2 = 220 \text{ V}$

rms current in the wire lines is given as:

$$I = \frac{P}{V_1}$$

$$= \frac{800 \times 10^3}{4000} = 200 \text{ A}$$

(a) Line power loss = $I^2 R$

$$= (200)^2 \times 15$$

$$= 600 \times 10^3 \text{ W}$$

$$= 600 \text{ kW}$$

(b) Assuming that the power loss is negligible due to the leakage of the current:

Total power supplied by the plant = 800 kW + 600 kW = 1400 kW

(c) Voltage drop in the power line = $IR = 200 \times 15 = 3000 \text{ V}$ Hence, total voltage transmitted from the plant = 3000 + 4000

= 7000 V

Also, the power generated is 440 V.

Hence, the rating of the step-up transformer situated at the power plant is 440 V – 7000 V.

Question 26:

Do the same exercise as above with the replacement of the earlier transformer by a 40,000-220 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Solution 26:

The rating of a step-down transformer is 40000 V–220 V.

Input voltage, $V_1 = 40000$ V

Output voltage, $V_2 = 220$ V

Total electric power required, $P = 800$ kW = 800×10^3 W

Source potential, $V = 220$ V

Voltage at which the electric plant generates power, $V' = 440$ V

Distance between the town and power generating station, $d = 15$ km

Resistance of the two wire lines carrying power = $0.5 \Omega/\text{km}$

Total resistance of the wire lines, $R = (15 + 15)0.5 = 15 \Omega$

$P = V_1 I$

Rms current in the wire line is given as:

$$I = \frac{P}{V_1}$$

$$= \frac{800 \times 10^3}{40000} = 20 \text{ A}$$

(a) Line power loss = $I^2 R$

$$= (20)^2 \times 15$$

$$= 6 \text{ kW}$$

(b) Assuming that the power loss is negligible due to the leakage of current.

Hence, power supplied by the plant = $800 \text{ kW} + 6 \text{ kW} = 806 \text{ kW}$

(a) Voltage drop in the power line = $IR = 20 \times 15 = 300$ V

(b) Hence, voltage that is transmitted by the power plant

$$= 300 + 40000 = 40300 \text{ V}$$

The power is being generated in the plant at 440 V. Hence, the rating of the step-up transformer

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needed at the plant is 440 V – 40300 V.

Hence, power loss during transmission = $\frac{600}{1400} \times 100 = 42.8\%$

In the previous exercise, the power loss due to the same reason is = $\frac{6}{806} \times 100 = 0.744\%$

Since the power loss is less for a high voltage transmission, high voltage transmissions are preferred for this purpose.
