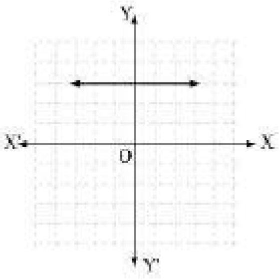
EXERCISE NO: 2.1

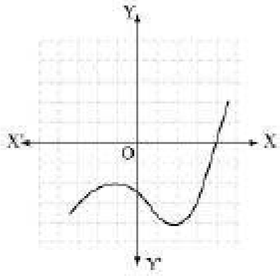
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some Polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

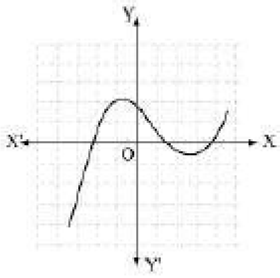
(i)



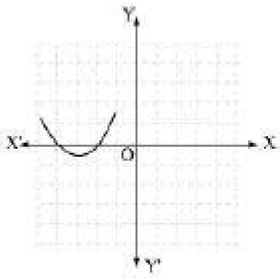
(ii)



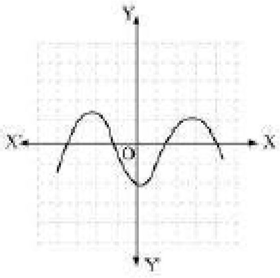
(iii)



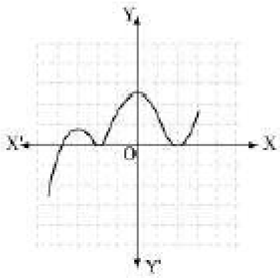
(iv)



(v)



(vi)



Solution 1:

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

EXERCISE NO: 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) x^2-2x-8	(ii) $4s^2-4s+1$	(iii) $6x^2-3-7x$
(iv) $4u^2+8u$	(v) t^2-15	(vi) $3x^2-x-4$

Solution 1:

(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when $x-4=0$ or $x+2=0$, i.e., when $x=4$ or $x=-2$

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{(-2)}{1}=\frac{-(\text{Coefficient of }x)}{\text{Coefficient of }x^2}$

Product of zeroes $=4x(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of }x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2-4s+1$ is zero when $2s-1=0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2-4s+1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1\frac{(-4)}{4}=\frac{-(\text{Coefficient of }s)}{\text{Coefficient of }s^2}$

Product of zeroes $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of }s^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero when $3x+1=0$ or $2x-3=0$, i.e.,

$x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2-3-7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u = 4u^2 + 8u + 0$

$= 4u(u + 2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and −2.

Sum of zeroes $= 0 + (-2) = -2 = \frac{(-8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes $= 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v)

$t^2 - 15$

$= t^2 = 0t - 15$

$= (t - \sqrt{15})(t + \sqrt{15})$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are and $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$

Product of zeroes $= (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2 - x - 4$

The value of $3x2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e., when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and −1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}+(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) 0, 5 (iv) 1, 1 (v) $-\frac{1}{4},\frac{1}{4}$

(vi) 4, 1

Solution 2:

(i) $\frac{1}{4},-1$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$$

$$\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$$

If a = 4, then b = −1, c = -4

Therefore, the quadratic polynomial is $4x^2-x-4$.

(ii) $\sqrt{2},\frac{1}{3}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\sqrt{2}=\frac{3\sqrt{2}}{3}=\frac{-b}{a}$$

$$\alpha\beta=\frac{1}{3}=\frac{c}{a}$$

If a = 3, then b = $-3\sqrt{2}$, c = 1

Therefore, the quadratic polynomial is $3x^2-3\sqrt{2}x+1$.

(iii) 0, $\sqrt{5}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=0=\frac{0}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=\sqrt{5}=\frac{\sqrt{5}}{1}=\frac{c}{a}$$

If a = 1, then b = 0, c = $\sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$

If a = 1, then b = -1, c = 1

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4},\frac{1}{4}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{-1}{4}=\frac{-b}{a}$$

$$\alpha\times\beta=\frac{1}{4}=\frac{c}{a}$$

If a = 4, then b = 1, c = 1

Therefore, the quadratic polynomial is $4x^2+x+1$.

(vi) 4, 1

Let the polynomial be ax^2+bx+c.

$$\alpha+\beta=4=\frac{4}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$

If a = 1, then b = −4, c =1

Therefore, the quadratic polynomial is x^2-4x+1.

EXERCISE NO: 2.3

Question 1:

Divide the polynomial *p*(*x*) by the polynomial *g*(*x*) and find the quotient and remainder in each of the following:

(i) $p(x)=x^3-3x^2+5x-3,\ g(x)=x^2-2$

(ii) $p(x)=x^4-3x^2+4x+5,\ \ g(x)=x^2+1-x$

(iii) $p(x)=x^4-5x+6,\ g(x)=2-x^2$

Solution 1:

(i)

$p(x)=x^3-3x^2+5x-3,$

$g(x)=x^2-2$

$$
\begin{array}{r|l}
 & \qquad\qquad\qquad x-3 \\ \hline
x^2-2 & x^3-3x^2+5x-3 \\
 & x^3 \qquad\quad -2x \\
 & - \qquad\quad\ + \\ \hline
 & \quad -3x^2+7x-3 \\
 & \quad -3x^2 \qquad +6 \\
 & \quad\ \ + \qquad\quad - \\ \hline
 & \qquad\qquad 7x-9 \\ \hline
\end{array}
$$

Quotient = x – 3

Remainder = 7x – 9

(ii) $p(x)=x^4-3x^2+4x+5=x^4+0.x^3-3x^2+4x+5$

$g(x)=x^2+1-x=x^2-x+1$

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 \qquad - 3x^2 + 4x - 5 \\
 & x^4 - x^3 \quad + x^2 \\
 & - \quad + \qquad - \\
\cline{2-2}
 & \qquad x^3 \; -4x^2 + 4x + 5 \\
 & \qquad x^3 - x^2 \; + x \\
 & \qquad - \quad + \quad - \\
\cline{2-2}
 & \qquad\qquad -3x^2 + 3x + 5 \\
 & \qquad\qquad -3x^2 + 3x - 3 \\
 & \qquad\qquad + \quad - \quad + \\
\cline{2-2}
 & \qquad\qquad\qquad 8
\end{array}
$$

Quotient = $x^2 + x - 3$
Remainder = 8

(iii)

$p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$
\begin{array}{r|l}
 & \qquad\quad -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0.x^2 - 5x + 6 \\
 & x^4 - 2x^2 \\
 & - \quad + \\
\cline{2-2}
 & \quad 2x^2 - 5x + 6 \\
 & \quad 2x^2 \; - 4 \\
 & \quad - \qquad + \\
\cline{2-2}
 & \qquad - 5x + 10 \\
\cline{2-2}
\end{array}
$$

Quotient = $-x^2 - 2$
Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2-3, 2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

(iii) $x^2-3x+1, x^5-4x^3+x^2+3x+1$

Solution 2:

(i) $t^2-3, 2t^4+3t^3-2t^2-9t-12$

$t^2-3=t^2+0.t-3$

$$\begin{array}{r|l}
 & \qquad\qquad\quad 2t^2+3t+4 \\ \cline{2-2}
t^2+0.t2-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4+0.t^3-6t^2 \\
 & - \quad - \quad + \\ \cline{2-2}
 & \quad 2t^3+4t^2-9t-12 \\
 & \quad 3t^3+0.t^2-9t \\
 & \quad - \quad - \quad + \\ \cline{2-2}
 & \quad 4t^2+0.t-12 \\
 & \quad 4t^2+0.t-12 \\ \cline{2-2}
 & \qquad 0 \\ \cline{2-2}
\end{array}$$

Since the remainder is 0,

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

$$\begin{array}{r|l} & 3x^2+4x+2 \\ \hline x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\ & \underset{-}{}3x^4 \underset{-}{+}9x^3 \underset{+}{-}3x^2 \\ \hline & -4x^3-10x^2+2x+2 \\ & \underset{+}{-}4x^3 \underset{+}{-}12x^2 \underset{+}{-}4x \\ \hline & 2x^2+6x+2 \\ & \underset{-}{}2x^2 \underset{-}{+}6x \underset{-}{+}2 \\ \hline & \text{x} \quad \text{x} \quad \text{x} \end{array}$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$

(iii) $x^2-3x+1, x^5-4x^3+x^2+3x+1$

$$\begin{array}{r|l} & x^2-1 \\ \hline x^2-3x+1 & x^5-4x^3+x^2+3x+1 \\ & \underset{-}{}x^5 \underset{+}{-}3x^3 \underset{-}{+}x^2 \\ \hline & -x^3 \qquad +3x+1 \\ & \underset{+}{-}x^3 \qquad \underset{-}{+}3x \underset{+}{-}1 \\ \hline & \qquad\qquad 2 \end{array}$$

Since the remainder $\neq 0$,

$x^2-3x+1, x^5-4x^3+x^2+3x+1$

Question 3:

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Solution 3:

$p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\sqrt{\frac{5}{3}}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$

$$\begin{array}{ll}
 & \qquad\qquad\qquad\qquad 3x^2+6x+3 \\
x^2+0.x-\frac{5}{3} & \overline{\big)\, 3x^4+6x^3-2x^2-10x-5} \\
 & \;\; 3x^4+0.x^3-5x^2 \\
 & \;\; - \qquad - \qquad + \\
\hline
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3+0x^2-10x \\
 & \qquad - \qquad - \qquad + \\
\hline
 & \qquad\; 3x^2+0x-5 \\
 & \qquad\; 3x^2+0x-5 \\
 & \qquad - \qquad - \qquad + \\
\hline
 & \qquad\quad 0 \\
\hline
\end{array}$$

$$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$$

$$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x+1=0$

$x=-1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1.

Question 4:

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Solution 4:

$p(x) = x^3 - 3x^2 + x + 2$ (Dividend)

g(x) = ? (Divisor)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) x (x-2) + (-2x+4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x-2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x-2)$$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$

$$\begin{array}{r|l}
 & x^2 - x + 1 \\ \hline
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad\;\; + \\ \cline{2-2}
 & \quad -x^2 + 3x - 2 \\
 & \quad -x^2 + 2x \\
 & \qquad + \quad\; - \\ \cline{2-2}
 & \qquad x - 2 \\
 & \qquad x - 2 \\
 & \qquad - \quad + \\ \cline{2-2}
 & \qquad\; 0 \\ \cline{2-2}
\end{array}$$

$$\therefore g(x) = \left(x^2 - x + 1\right)$$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

Solution 5:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.
Here, $p(x) = 6x^2 + 2x + 2$
$g(x) = 2$
$q(x) = 3x^2 + x + 1$ and $r(x) = 0$
Degree of $p(x)$ and $q(x)$ is the same i.e., 2.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$

$$6x^2 + 2x + 2 = 2\left(3x^2 + x + 1\right)$$

$$= 6x^2 + 2x + 2$$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$
Let us assume the division of $x^3 + x$ by x^2,
Here, $p(x) = x^3 + x$
$g(x) = x^2$
$q(x) = x$ and $r(x) = x$
Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
$x^3 + x = (x^2) \times x + x$
$x^3 + x = x^3 + x$
Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$
Degree of remainder will be 0 when remainder comes to a constant.
Let us assume the division of $x^3 + 1$ by x^2.
Here, $p(x) = x^3 + 1$
$g(x) = x^2$
$q(x) = x$ and $r(x) = 1$
Clearly, the degree of $r(x)$ is 0.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
$x^3 + 1 = (x^2) \times x + 1$
$x^3 + 1 = x^3 + 1$
Thus, the division algorithm is satisfied.

EXERCISE NO: 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2; \frac{1}{2},1,2$ $2x^3 + x^2 – 5x +2$; ½, 1, -2

(ii) $x^3-4x^2+5x-2; 2,1,1$

Solution 1:

(i)$p(x) = 2x^3+x^2-5x+2$

Zeroes for this polynomial are $\frac{1}{2},1,-2$

$$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$$
$$=\frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$$
$$=0$$
$$p(1)=2\times1^3+1^2-5\times1+2$$
$$=0$$
$$p(-2)=2(-2)^3+(-2)^2-5(-2)+2$$
$$=-16+4+10+2=0$$

Therefore, $\frac{1}{2}$, 1, and −2 are the zeroes of the given polynomial.

Comparing the given polynomial with ax^3+bx^2+cx+d, we obtain $a = 2$, $b = 1$, $c = -5$, $d = 2$

We can take $\alpha=\frac{1}{2}, \beta=1, y=-2$

$$\alpha+\beta+\gamma=\frac{1}{2}+1+(-2)=-\frac{1}{2}=\frac{-b}{a}$$
$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{1}{2}\times1+1(-2)+\frac{1}{2}(-2)=\frac{-5}{2}=\frac{c}{a}$$
$$\alpha\beta\gamma=\frac{1}{2}\times1\times(-2)=\frac{-1}{1}=\frac{-(2)}{2}=\frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x)=x^3-4x^2+5x-2$

Zeroes for this polynomial are 2, 1, 1

$p(2)=2^3-4(2^2)+5(2)-2$

$=8-16+10-2=0$

$p(1)=1^3-4(1^2)+5(1)-2$

$=1-4+5-2=0$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with ax^3+bx^2+cx+d, we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.

Verification of the relationship between zeroes and coefficient of the given polynomial

Sum of zeroes $=2+1+1=4=\frac{-(-4)}{1}=\frac{-b}{a}$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1)

$=2+1+2=5=\frac{(5)}{1}=\frac{c}{a}$

Multiplication of zeroes = $2 \times 1 \times 1 = 2=\frac{-(-2)}{1}=\frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Solution 2:

Let the polynomial be ax^3+bx^2+cx+d and the zeroes be $\alpha, \beta,$ and γ.

It is given that

$\alpha+\beta+\gamma=\frac{2}{1}=\frac{-b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$

If *a* = 1, then *b* = −2, *c* = −7, *d* = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$are $a - b, a, a + b$, find *a* and *b*.

Solution 3:

$p(x) = x^3 - 3x^2 + x + 1$

Zeroes are *a* − *b*, *a* + *a* + *b*

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

p = 1, *q* = −3, *r* = 1, *t* = 1

Sum of zeroes = a – b + a + a + b

$\frac{-q}{p} = 3a$

$\frac{-(-3)}{1} = 3a$

$3 = 3a$

$a = 1$

The zeroes are 1 – b, 1 + b.

Multiplication of zeroes = 1(1 – b)(1 + b)

$\frac{-t}{p} = 1 - b^2$

$\frac{-1}{1} = 1 - b^2$

$1 - b^2 = -1$

$1 + 1 = b^2$

$b = \pm\sqrt{2}$

Hence, *a* = 1 and *b* = $\sqrt{2}$ or$-\sqrt{2}$.

Question 4:

It two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$, find other zeroes.

Solution 4:

Given that $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)=x^2+4-4x-3$

$=x^2-4x+1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4-6x^3-26x^2+138x-35$ by x^2-4x+1.

$$
\begin{array}{rl}
 & \qquad\qquad\qquad\quad x^2-2x-35 \\
x^2-4x+1 \big) & \overline{x^4-6x^3-26x^2+138x-35} \\
 & x^4-4x^3+x^2 \\
 & \underline{-\quad\;\; +\quad\;\; -\quad} \\
 & \quad -2x^3-27x^2+138x-35 \\
 & \quad -2x^3+8x^2-2x \\
 & \quad \underline{+\qquad -\qquad +} \\
 & \quad -35x^2+140x-35 \\
 & \quad -35x^2+140x-35 \\
 & \qquad \underline{+\qquad -\qquad +} \\
 & \qquad\qquad \underline{\quad 0 \quad}
\end{array}
$$

Clearly, $=x^4-6x^3-26x^2+138x-35=\left(x^2-4x+1\right)\left(x^2-2x-35\right)$

It can be observed that $\left(x^2-2x-35\right)$ is also a factor of the given polynomial.

And $=\left(x^2-2x-35\right)=(x-7)(x+5)$

Therefore, the value of the polynomial is also zero when or $x-7=0$

Or $x+5=0$

Or $x=7$ or -5

Hence, 7 and −5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x - 10$ is divided by another Polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Solution 5:

By division algorithm,
Dividend = Divisor × Quotient + Remainder
Dividend − Remainder = Divisor × Quotient
$x^4 - 6x^3 + 16x^2 - 25x - 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.
Let us divide by $x^4 - 6x^3 + 16x^2 - 26x - 10 - a$ by $x^2 - 2x + k$

$$x^2 - 4x + (8 - k)$$

$$x^2 - 2x + k \overline{\big)\, x^4 - 6x^3 + 16x^2 - 26x + 10 - a}$$

$$x^4 - 2x^3 + kx^2$$

$$\underline{- \quad + \quad -}$$

$$-4x^3 + (16 - k)x^2 - 26x$$

$$-4x^3 + \quad 8x^2 - 4kx$$

$$\underline{+ \quad - \quad +}$$

$$(8 - k)x^2 - (26 - 4k)x + 10 - a$$

$$(8 - k)x^2 - (16 - 2k)x + (8k - k^2)$$

$$\underline{- \quad + \quad -}$$

$$\underline{(-10 + 2k)x + (10 - a - 8k + k^2)}$$

$$(x^2 - 4x + 1)(x^2 - 2x - 35) = (x - 7)(x + 5)$$

It can be observed that $(-10 + 2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10 + 2k) = 0$ and $(10 - a - 8k + k^2) = 0$

For $(-10 + 2k) = 0$,

$2k = 10$

And thus, $k = 5$

For $\left(10 - a - 8k + k^2\right) = 0$

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$
