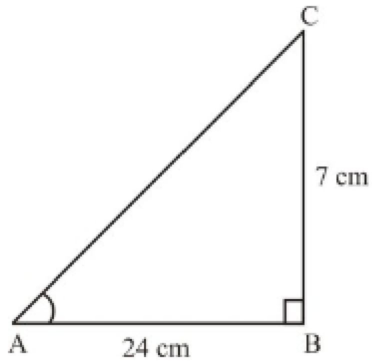
Exercise (8.1)

Question 1:

In ΔABC right angled at B, AB = 24 cm, BC = 7 cm. Determine
(i) sin A, cos A
(ii) sin C, cos C

Solution 1:

Let us draw a right angled triangle ABC



Given,

- AB = 24 cm
- BC = 7 cm
- Sin A = ?
- Cos A= ?
- Sin C = ?
- Cos C = ?
- AC = ?

We know that by Pythagoras theorem for ΔABC,
$AC^2 = AB^2 + BC^2$
$= 24^2 + 7^2$ (By Substituting the values)
= 576 + 49
$= 625 \text{ cm}^2$
∴ Hypotenuse, AC = 25 cm

$$\text{(i) } \sin A = \frac{\text{Side opposite to} \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

$$\text{(ii) } \cos A = \frac{\text{Side adjacent to} \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

$$\text{(iii) } \sin C = \frac{\text{Side opposite to} \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$$

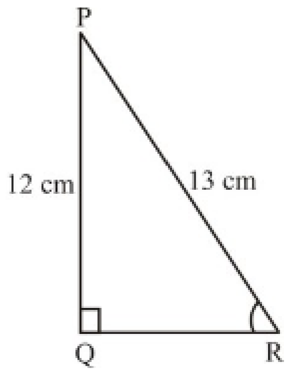
$$= \frac{24}{25}$$

$$\text{(iv) } \cos C = \frac{\text{Side adjacent to} \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

Question 2:

In the given figure find tan P − cot R



Solution 2:

From the above Figure,

Given

- PQ= 12 cm
- PR = 13 cm
- QP= ?
- tan P – cot R = ?

We know that by applying Pythagoras theorem for ΔPQR,

$PR^2 = PQ^2 + QR^2$

$13^2 = 12^2 + QR^2$ (By Substituting the values)

$169 = 144 + QR^2$

$QR^{2 =} 169 - 144$

$QR^2 = 25\ cm^2$

QR = 5 cm

Hence,

$$\tan\ P = \frac{\text{Side opposite to} \angle P}{\text{Side adjacent to} \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to} \angle R}{\text{Side opposite to} \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

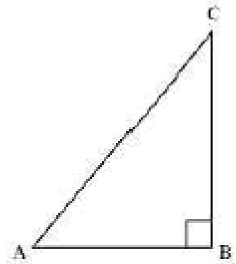
$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

Question 3:

If $\sin\ A = \frac{3}{4}$, calculate cos A and tan A.

Solution 3:

From the figure,



Let ΔABC is a right-angled triangle
Given,

- $\sin A = \frac{3}{4}$,
    - We know that Sine = $\frac{\text{Opposite}}{\text{Hypotenuse}}$
        - Hence with respect to angle A,
            - BC = 3
            - AC= 4
            - AB = ?
- cos A = ?
- tan A = ?

By Applying Pythagoras theorem in ΔABC, we get
$AC^2 = AB^2 + BC^2$
$4^2 = AB^2 + 3^2$
$16 - 9 = AB^2$
$AB^2 = 7$
$AB = \sqrt{7}$

$$\cos A = \frac{\text{Side adjacent to} \angle A}{\text{Hypotenuse}}$$

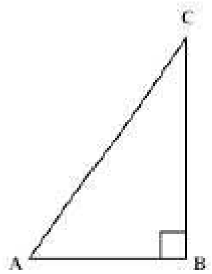
$$\cos A = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to} \angle A}{\text{Side adjacent to} \angle A}$$

$$\text{Tan} = \frac{3}{\sqrt{7}}$$

Question 4:
Given 15 cot A = 8. Find sin A and sec A.

Solution 4:
From the Figure,



Let ABC be the right-angled triangle,
Given,

- 15 cot A = 8
- Sin A = ?
- Sec A = ?

$\cot A = \frac{\text{Side adjacent to} \angle A}{\text{Side opposite to} \angle A}$

$= \frac{AB}{BC}$

From Given,
15 cot A = 8

$\text{Cot } A = \frac{8}{15}$ (By Transposing)

$\frac{AB}{BC} = \frac{8}{15}$

By applying Pythagoras theorem in ΔABC, we get

$AC^2 = AB^2 + BC^2$

$= 8^2 + 15^2$

$= 64^2 + 225^2$

$= 289^2$

AC = 17

$\sin A = \frac{\text{Side opposite to} \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{15}{17}$

$\sec A = \frac{\text{Hypotenuse}}{\text{Side opposite to} \angle A} = \frac{AC}{AB}$

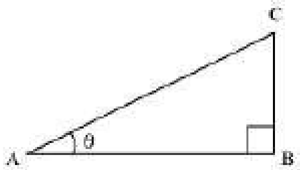
$= \frac{17}{8}$

Question 5:

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Solution 5:

From the Figure,



Let ΔABC be a right-angle triangle
Given,

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to}\angle\theta}$$

$$= \frac{13}{12} = \frac{AC}{AB}$$

Hence

- AC = 13
- AB = 12
- BC = ?
- Sin θ = ?
- Cos θ = ?
- Tan θ = ?
- Cot θ = ?
- Cosec θ = ?

By applying Pythagoras theorem in ΔABC, we obtain
$(AC)^2 = (AB)^2 + (BC)^2$
$13^2 = 12^2 + BC^2$
$169 = 144 + BC^2$
$25 = BC^2$
BC = 5

$$\sin\theta = \frac{\text{Side opposite to}\angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12}{13}$$

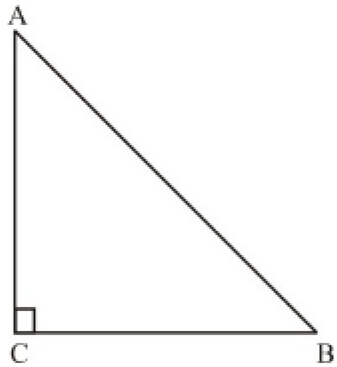
$$\tan\theta = \frac{\text{Side opposite to}\angle\theta}{\text{Side adjacent to}\angle\theta} = \frac{BC}{AB} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Side opposite to}\angle\theta} = \frac{AB}{BC} = \frac{12}{5}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to}\angle\theta} = \frac{AC}{BC} = \frac{13}{5}$$

Question 6:
If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Solution 6:
From the Figure,



Given

- Let ΔABC be a right Angled Triangle
- ∠A and ∠B are Acute Angles
- Cos A = Cos B

To Prove:

∠A = ∠B

Proof:

In the Right Angled Triangle ABC,

$\cos A = \frac{\text{Side adjacent to} \angle A}{\text{Hypotenuse}}$ = AC/ AB

$\cos B = \frac{\text{Side adjacent to} \angle B}{\text{Hypotenuse}}$ = BC/ AB

Since we know Cos A = Cos B

AC/ AB = BC/ AB

Hence by observation,

AC = AB

Hence, ∠A = ∠B (Angles opposite to the equal sides of the triangle).

Question 7:

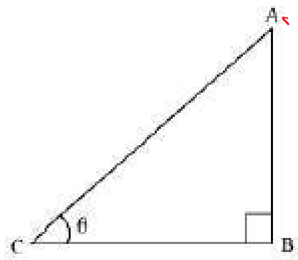
If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Solution 7:

From the Figure,



Given,
Let ΔABC be a right triangle ABC,

$$\cot\theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Side opposite to}\angle\theta} = \frac{BC}{AB} = \frac{7}{8}$$

Hence,

- BC = 7
- AB = 8
- AC = ?

By applying Pythagoras theorem in ΔABC, we obtain
$AC^2 = AB^2 + BC^2$
$= 8^2 + 7^2$
= 64 + 49
= 113
By Taking the Square roots,
AC = $\sqrt{113}$

$$\sin\theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{\sqrt{113}}$$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

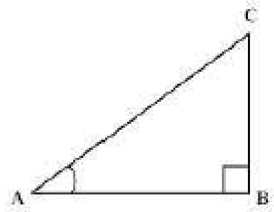
$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $$\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Question 8:

If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not

Solution 8:
From the figure,



Given,
Let ΔABC be a right angled triangle.

- 3cot A = 4

Hence, cot A = $\frac{4}{3}$

We know that,

$$\cot A = \frac{\text{Side adjacent to} \angle A}{\text{Side opposite to} \angle A} = \frac{AB}{BC} = \frac{4}{3}$$

Hence, AB = and BC = 3. AC = ?
By applying the Pythagoras Theorem in ΔABC,
$(AC)^2 = (AB)^2 + (BC)^2$
$= 4^2 + 3^2$
$= 16 + 9$
$= 25$
AC = 5

$$\cos A = \frac{\text{Side adjacent to} \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to} \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to} \angle A}{\text{Side adjacent to} \angle A} = \frac{BC}{AB} = \frac{3}{4}$$

By substituting the above values of trigonometric functions in the LHS of the Equation,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

By substituting the above values of trigonometric functions in the RHS of the Equation

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 a$$

Hence it is proved.

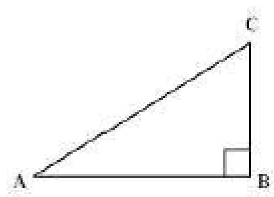
Question 9:

In ABC, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i) Sin A cos C + cos A sin C
(ii) Cos A cos C − sin A sin C

Solution 9:

From the figure,



Given,
Let ΔABC be a right angled triangle

- $\tan A = \frac{1}{\sqrt{3}}$
  $\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Hence,

- BC = 1
- AB = $\sqrt{3}$
- AC = ?

By applying Pythagoras Theorem for ΔABC,
$AC^2 = AB^2 + BC^2$
$= (\sqrt{3})^2 + 1^2$
$= 3 + 1 = 4$
AC = 2

$$\sin A = \frac{\text{Side opposite to} \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to} \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to} \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

By substituting the values of the trigonometric functions below in the equation below,

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) cos A cos C − sin A sin C

By substituting the values of the trigonometric functions below in the equation below,

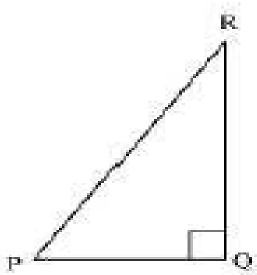
$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Question 10:

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Solution 10:

From the figure,



Given

Let ΔPQR be a right angled triangle

- PR + QR = 25cm
- PQ = 5cm
- sin P = ?
- cos P = ?
- tan P = ?
- PR = ?

Therefore, QR = 25 − x

By applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = 5^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

Hence, $x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

By Substituting the values of the obtained above in the trigonometric functions below,

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A $= \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

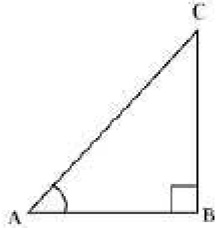
(v) sin θ $= \frac{4}{3}$, for some angle θ

Solution 11:

(i) False, because sides of a right angled triangle may have any length, So tan A may have any value.

(ii) $\sec A = \frac{12}{5}$

True, as the value of Sec A > 1,



$$\sec A = \frac{1}{\cos A} = \frac{1}{\frac{\text{Side of Adjacent } \angle A}{\text{Hypotenuse}}} = \frac{\text{Hypotenuse}}{\text{Side of Adjacent } \angle A}$$

As Hypotenuse is the largest Side, Sec A> 1

(iii) Abbreviation used for cosecant of ∠A is cosec A. And cos A is the abbreviation used for cosine of ∠A. Hence, the given statement is false.

(iv) Cot A is not the product of cot and A. It is the cotangent of ∠A. ‘Cot’ separated from ‘A’ has no meaning. Hence, the given statement is false.

(v) Sin θ $= \frac{4}{3}$

We know that in a right-angled triangle,

$\sin\theta = \dfrac{\text{Side opposite to}\angle\theta}{\text{Hypotenuse}}$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Also, the value of Sine should be less than 1 always. Therefore, such value of sin θ is not possible. Hence, the given statement is false

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Exercise (8.2)

Question 1:

Evaluate the following

(i) Sin60° cos30° + sin30° cos 60°

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ - \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 + 30^\circ - \tan^2 45^\circ}{\sec^2 30^\circ + \cos^2 30^\circ}$

Solution 1:

We know that,

Exact Values of Trigonometric Functions				
Angle (θ)		sin(θ)	cos(θ)	tan(θ)
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not Defined

(i) sin60° cos30° + sin30° cos 60°

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ (By Substituting the Values taken from the chart above)

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

(By Substituting the Values taken from the chart above)

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

(By Substituting the Values taken from the chart above)

– –

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2 + 2\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2} \times 2 + (\sqrt{3} + 1)}$$

$$= \frac{\sqrt{3}}{\sqrt{2} \times 2 + (\sqrt{3} + 1)} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

(By multiplying & dividing by $\sqrt{3} - 1$)

$$= \frac{\sqrt{3}(\sqrt{3} - 1)}{\sqrt{2} \times 2 + (\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{3 - \sqrt{3}}{2\sqrt{2}((\sqrt{3})^2 - 1^2)}$$

$$= \frac{3 - \sqrt{3}}{2\sqrt{2}(3 - 1)} = \frac{3 - \sqrt{3}}{4\sqrt{2}}$$

(iv)

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

(By Substituting the Values taken from the chart above)

$$= \frac{\frac{3\sqrt{3} - 4}{2\sqrt{3}}}{\frac{3\sqrt{3} + 4}{2\sqrt{3}}} = \frac{(3\sqrt{3} - 4)}{(3\sqrt{3} - 4)}$$

$$= \frac{(3\sqrt{3} - 4)(3\sqrt{3} - 4)}{(3\sqrt{3} + 4)(3\sqrt{3} - 4)} = \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2}$$

(By Using $(a + b)(a - b) = a^2 - b^2$)

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}$$

(v)

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

(By Substituting the Values taken from the chart above)

$$= \frac{5\left(\frac{1}{2}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15 + 64 - 12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice.

(i) $\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ} =$ ___________

(A). sin60°
(B). cos60°
(C). tan60°
(D). sin30°

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$ ___________

(A). tan90°
(B). 1
(C). sin45°
(D). 0

(iii) sin 2A = 2sin A is true when A = ___________

(A). 0°
(B). 30°
(C). 45°
(D). 60°

(iv) $\frac{2\tan 30^\circ}{1 - \tan^2 30^\circ} =$ ___________

(A). cos60°
(B). sin60°
(C). tan60°
(D). sin30°

Solution 2:

We know that,

Exact Values of Trigonometric Functions				
Angle ( )		sin( )	cos( )	tan( )
Degrees	Radians			
0°	0	0	1	0
30°				
45°				1
60°				
90°		1	0	Not Defined

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$ ____________

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$ (By Substituting the Values taken from the chart above)

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$ ____________

$$\frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$ (By Substituting the Values taken from the chart above)

Hence, (D) is correct.

(iii) Out of the given alternatives, only A = 0° is correct.
As sin 2A = sin 0° = 0 (By Substituting the Values taken from the chart above)
2 sin = 2sin 0° = 2(0) = 0
Hence, (A) is correct.

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$ ____________

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$ (By Substituting the Values taken from the chart above)

$= \sqrt{3}$

Out of the given alternatives, only tan 60° = $\sqrt{3}$

Hence, (C) is correct.

Question 3:

If and; $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$

$0° < A + B \leq 90°$, $A > B$ find A and B.

Solution 3:

We know that,

Exact Values of Trigonometric Functions				
Angle ( )		sin( )	cos( )	tan( )
Degrees	Radians			
0°	0	0	1	0
30°				
45°				1
60°				
90°		1	0	Not Defined

$\tan(A+B)=\sqrt{3}$

$\Rightarrow \tan(A+B)=\tan 60°$ (By Substituting the Values taken from the chart above)

$\Rightarrow A + B = 60°$ ……..Equation (1)

$\tan(A-B)=\frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B)=\tan 30°$ (By Substituting the Values taken from the chart above)

$\Rightarrow A - B = 30°$ …Equation (2)

On adding both Equation (1) & Equation (2), we obtain

$A + B + A – B = 60° + 30°$

$2A = 90°$

$\Rightarrow A = 45°$

By substituting the value of A in Equation (1), we obtain

$45° + B = 60°$

$B = 15°$

Therefore, ∠A = 45° and ∠B = 15°

Question 4:
State whether the following are true or false. Justify your answer.
(i) sin(A + B) = sin A + sin B
(ii) The value of sin θ increases as θ increases
(iii) The value of cos θ increases as θ increases
(iv) sin θ = cos θ for all values of θ
(v) Cot A is not defined for A = 0°

Solution 4:
We know that,

Exact Values of Trigonometric Functions				
Angle ( )		sin( )	cos( )	tan( )
Degrees	Radians			
0°	0	0	1	0
30°				
45°				1
60°				
90°		1	0	Not Defined

(i) sin (A + B) = sin A + sin B

- For the purpose of verification, Take A = 30° and B = 60°

By substituting the values in LHS,
sin (A + B) = sin (30° + 60°)
= sin 90°
= 1
By substituting the values in RHS,
sin A + sin B = sin 30° + sin 60°
$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$
Clearly, sin (A + B) ≠ sin A + sin B
Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90°
We know that

- sin 0° = 0
- $\sin 30° = \frac{1}{2} = 0.5$

- $\sin 45° = \frac{1}{\sqrt{2}} = 0.707$
- $\sin 60° = \frac{\sqrt{3}}{2} = 0.866$
- sin 90° = 1

Hence, the given statement is true.

(iii) We know that,

- cos 0° = 1
- $\cos 30° = \frac{\sqrt{3}}{2} = 0.866$
- $\cos 45° = \frac{1}{\sqrt{2}} = 0.707$
- $\cos 60° = \frac{1}{2} = 0.5$

It can be observed that the value of cos θ does not increase in the interval of 0° < θ < 90°. Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.
This is true when θ = 45°

As $\sin 45^\circ = \frac{1}{\sqrt{2}}$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ.

As $\sin 30^\circ = \frac{1}{2}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$,

Hence, the given statement is false.

(v) tan 0° = 0 and cot A is not defined for A = 0°

As, $\cot A = \frac{\cos A}{\sin A}$ and $\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0}$ undefined

Hence, the given statement is true.

Exercise (8.3)

Question 1:
Evaluate:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(III) cos 48° − sin 42°
(IV) cosec 31° − sec 59°

Solution 1:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin\left(90^\circ - 72^\circ\right)}{\cos 72^\circ}$ (Since Sin (90° − θ) = Cos θ)

$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan\left(90^\circ - 64^\circ\right)}{\cot 64^\circ}$ (Since tan (90° − θ) = Cot θ)

$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(III) cos 48° − sin 42°
= cos (90°− 42°) − sin 42°
= sin 42° − sin 42° (Since Sin (90° − θ)= Cos θ)
= 0

(IV) cosec 31° − sec 59°
= cosec (90° − 59°) − sec 59° (Since Cosec (90° − θ) = Sec θ)
= sec 59° − sec 59°
= 0

Question 2:
Show that
(I) tan 48° tan 23° tan 42° tan 67° = 1
(II) cos 38° cos 52° − sin 38° sin 52° = 0

Solution 2:
(I) tan 48° tan 23° tan 42° tan 67° = 1
Taking LHS,
tan 48° tan 23° tan 42° tan 67°------------Equation (1)
We know that tan (90° – A) = tan A
By manipulating the Equation (1) using the property above,

$= \tan(90° - 42°) \tan(90° - 67°) \tan 42° \tan 67°$

$= \cot 42° \cot 67° \tan 42° \tan 67°$

$= (\cot 42° \tan 42°)(\cot 67° \tan 67°)$ (By rearranging)

$= (1)(1)$ [As $\cot A. \tan A = 1$]

$= 1$

(II) $\cos 38° \cos 52° - \sin 38° \sin 52°$

Consider LHS :

$\cos 38° \cos 52° - \sin 38° \sin 52°$ --------------Equation (1)

$= \cos(90° - 52°) \cos(90° – 38°) - \sin 38° \sin 52°$ [As, $\cos(90 – \theta) = \sin \theta$]

$= \sin 52° \sin 38° - \sin 38° \sin 52°$

$= 0$

Question 3:
If $\tan 2A = \cot(A - 18°)$, where 2A is an acute angle, find the value of A.

Solution 3:
Given that,
$\tan 2A = \cot(A - 18°)$ -------------Equation (1)
We know that $\tan 2A = \cot(90 – 2A)$ by substituting this in Equation (1)
$\cot(90° - 2A) = \cot(A - 18°)$
Hence by Equating,
$90° - 2A = A - 18°$
$A + 2A = 90° + 18°$
$3A = 108°$
$A = 36°$

Question 4:
If $\tan A = \cot B$, prove that $A + B = 90°$

Solution 4:
Given,
$\tan A = \cot B$---------Equation (1),
We know that $\tan A = \cot(90 – A)$ by substituting this in Equation (1)
$\tan A = \tan(90° - B)$
By Equating,
$A = 90° - B$
$A + B = 90°$ (By Transposing)

Question 5:
If sec 4A = cosec (A − 20°), where 4A is an acute angle, find the value of A.

Solution 5:
Given,

sec 4A = cosec (A − 20°) ---------Equation (1),

We know that Sec A = Cosec (90-A) by substituting this in Equation (1)

cosec (90° − 4A) = cosec (A − 20°)

By Equating,

90° − 4A= A− 20°

110° = 5A (By Transposing)

A = 22°

Question 6:
If A, Band Care interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right)=\cos\frac{A}{2}$$

Solution 6:
We know that for a triangle ABC,
∠A + ∠B + ∠C = 180°
∠B + ∠C= 180° − ∠A (By Transposing)
Dividing both the sides by 2

$$\frac{\angle B+\angle C}{2}=90^\circ-\frac{\angle A}{2}$$

Applying Sine Angle on both the sides,

$$\sin\left(\frac{B+C}{2}\right)=\sin\left(90^\circ-\frac{A}{2}\right)$$

$$=\cos\left(\frac{A}{2}\right)$$

Question 7:
Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Solution 7:
Sin 67° + cos 75°
Since, Cos (90 – θ) = Sin θ and Sin (90 – θ) = Cos θ
= sin (90° − 23°) + cos (90° − 15°)
= cos 23° + sin 15°

Exercise (8.4)

Question 1:
Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Solution 1:
Consider a ΔABC with ∠B = 90°
Using the Trigonometric Identity,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1+\cot^2 A} \qquad \text{(By taking reciprocal both the sides)}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A} \qquad \left(\text{As } \frac{1}{\operatorname{cosec}^2 A} = \sin^2 A\right)$$

Therefore,

$$\sin A = \pm \frac{1}{\sqrt{1+\cot^2 A}}$$

For any sine value with respect to an angle in a triangle, sine value will never be negative. Since, sine value will be negative for all angles greater than 180°.

Therefore, $\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \frac{\sin A}{\cos A}$

However, Trigonometric Function, $\cot A = \frac{\cos A}{\sin A}$

Therefore, Trigonometric Function, $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$ (Trigonometric Identity)

$$= 1 + \frac{1}{\cos^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:
Write all the other trigonometric ratios of ∠A in terms of sec A.

Solution 2:
We know that,

Trigonometric Function, $\cos A = \frac{1}{\sec A}$ … Equation (1)

Also,
$\sin^2 A + \cos^2 A = 1$ (Trigonometric identity)
$\sin^2 A = 1 - \cos^2 A$ (By transposing)
Using value of Cos A from Equation (1) and simplifying further,

$$\sin A = \sqrt{1-\left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$ … Equation (2)

$\tan^2 A + 1 = \sec^2 A$ (Trigonometric identity)

$\tan^2 A = \sec^2 A - 1$ (By transposing)

Trigonometric Function,

$$\tan A = \sqrt{\sec^2 A - 1}$$ … Equation (3)

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$ … (By substituting Equations (1) and (2))

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$ … (By substituting Equation (2) and simplifying)

Question 3:

Evaluate

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) sin 25° cos 65° + cos 25° sin 65°

Solution 3:

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$$= \frac{\left[\sin\left(90^\circ - 27^\circ\right)\right]^2 + \sin^2 27^\circ}{\left[\cos\left(90^\circ - 73^\circ\right)\right]^2 + \cos^2 73^\circ}$$

$$= \frac{\left[\cos 27^\circ\right]^2 + \sin^2 27^\circ}{\left[\sin 73^\circ\right]^2 + \cos^2 73^\circ}$$ $(\sin(90^\circ - \theta) = \cos\theta \ \& \ \cos(90^\circ - \theta) = \sin\theta)$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

$$= \frac{1}{1}$$ (By Identity $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin25° cos65° + cos25° sin65°

$= (\sin 25^\circ)\{\cos(90^\circ - 25^\circ)\} + \cos 25^\circ\{\sin(90^\circ - 25^\circ)\}$ $(\sin(90^\circ - \theta) = \cos\theta$ & $\cos(90^\circ - \theta) = \sin\theta)$

$= (\sin 25^\circ)(\sin 25^\circ) + \cos 25^\circ(\cos 25^\circ)$

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1$ (By Identity $\sin^2 A + \cos^2 A = 1$)

Question 4:

Choose the correct option. Justify your choice.

(i) 9 sec 2A − 9 tan 2A = __________

(A) 1

(B) 9

(C) 8

(D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

(A) 0

(B) 1

(C) 2

(D) −1

(iii) (sec A + tan A) (1 – sin A) = __________

(A) sec A

(B) sin A

(C) cosec A

(D) cos A

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$

(A) sec 2A

(B) −1

(C) cot 2A

(D) tan 2A

Solution 4:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$ (By taking 9 as common)

$= 9(1)$ [By the identity, $1 + \sec^2 A = \tan^2 A$, Hence $\sec^2 A - \tan^2 A = 1$]

$= 9$

Hence, alternative (B) is correct.

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ) -------------- Equation (1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\text{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1),

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right) \qquad \text{(By taking LCM and multiplying)}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta} \qquad \text{(Using } a^2 - b^2 = (a + b)(a - b)\text{)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} \qquad \text{(Using identify } \sin^2\theta + \cos^2\theta = 1\text{)}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

Hence, alternative (C) is correct.

(iii) (sec A + tan A) (1 – sin A) ---------Equation (1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} \qquad \text{(By identify } \sin^2\theta + \cos^2\theta = 1\text{, Hence } 1 - \sin^2\theta = \cos^2\theta\text{)}$$

= cos A

Hence, alternative (D) is correct.

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

We know that the trigonometric functions,

$$\tan(x)=\frac{\sin(x)}{\cos(x)}$$

$$\cot(x)=\frac{\cos(x)}{\sin(x)}=\frac{1}{\tan(x)}$$

By substituting the above function in Equation (1),

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}$$

$$=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec}A+\cot A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2$

Solution 5:

(i) $\left(\operatorname{cosec}\theta-\cot\theta\right)^2=\frac{1-\cos\theta}{1+\cos\theta}$

L.H.S = $\left(\operatorname{cosec}\theta-\cot\theta\right)^2$ ----------Equation (1)

We know that the trigonometric functions,

$$\cot(x)=\frac{\cos(x)}{\sin(x)}=\frac{1}{\tan(x)}$$

$$\operatorname{cosec}(x)=\frac{1}{\sin(x)}$$

By substituting the above function in Equation (1),

$$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$$=\frac{\left(1-\cos\theta\right)^2}{\left(\sin\theta\right)^2}=\frac{\left(1-\cos\theta\right)^2}{\sin^2\theta}$$ (By Identity $\sin^2A + \cos^2A = 1$ Hence, $1 - \cos^2A = \sin^2A$)

$$=\frac{\left(1-\cos\theta\right)^2}{1-\cos^2\theta}$$

$$=\frac{\left(1-\cos\theta\right)^2}{(1-\cos\theta)(1+\cos\theta)}$$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$$=\frac{1-\cos\theta}{1+\cos\theta}$$

= RHS

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

L.H.S = $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$

$$=\frac{\cos^2 A+\left(1+\sin\right)^2}{\left(1+\sin\right)\left(\cos A\right)}$$ (Taking LCM and common denominator)

$$=\frac{\cos^2 A+1+\sin^2 A+2\sin A}{\left(1+\sin A\right)\left(\cos A\right)}$$

$$=\frac{\sin^2 A+\cos^2 A+1+2\sin A}{(1+\sin A)(\cos A)}$$

(By Identity $\sin^2 A + \cos^2 A = 1$)

$$=\frac{1+1+2\sin A}{(1+\sin A)(\cos A)}=\frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

By taking 2 common and simplifying

$$=\frac{2(1+\sin A)}{(1+\sin A)(\cos A)}=\frac{2}{\cos A}=2\sec A$$

$$=\text{R.H.S}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

LHS= $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$ ----------Equation (1)

We know that the trigonometric functions,

$$\tan(x)=\frac{\sin(x)}{\cos(x)}$$

$$\cot(x)=\frac{\cos(x)}{\sin(x)}=\frac{1}{\tan(x)}$$

By substituting the above function in Equation (1),

$$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\cos\theta}{\sin\theta}}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$ (By taking LCM and Common denominators)

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

Taking $\dfrac{1}{(\sin\theta-\cos\theta)}$ as common

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$,

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$=\frac{\left(1+\sin\theta\cos\theta\right)}{\left(\sin\theta\cos\theta\right)}$$ (By Identity $\sin^2 A + \cos^2 A = 1$)

= 1 + sec θ cosec θ

= R.H.S.

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

L.H.S = $\frac{1+\sec A}{\sec A}$ --------- Equation (1)

We know that the trigonometric functions,

$\sec(x)=\frac{1}{\cos(x)}$

By substituting the above function in Equation (1),

$$=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$=\frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}=\left(\cos A+1\right)$$

By taking 1= 1 – Cos A in both denominator and numerator

$$=\frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

By Identity $\sin^2 A + \cos^2 A = 1$

$$=\frac{1-\cos^2 A}{1-\cos A}=\frac{\sin^2 A}{1-\cos A}$$

= R.H.S

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$

Using the identity $\operatorname{cosec}^2 A=1+\cot^2 A$

L.H.S = $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Diving both numerator and denominator by Sin A

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$$

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

We know that, 1 + $\cot^2$ A = Cosec^2 A

Hence substituting 1= $\cot^2$ A - Cosec^2 A in the equation below

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 + \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A - \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

= cosec A + cot A

= R.H.S

(vi) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

L.H.S = $\sqrt{\frac{1 + \sin A}{1 - \sin A}}$ ------------Equation (1)

Multiplying and dividing by $\sqrt{(1 + \sin A)}$

$$\sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

Using $a^2 - b^2 = (a - b)(a + b)$,

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \sec A + \tan A$$ (By separating the denominators)

$$= \text{R.H.S}$$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta$

L.H.S = $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

Taking Sin θ and Cos θ common in both numerator and denominator respectively.

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

By Identity $\sin^2 A + \cos^2 A = 1$ hence, $\cos^2 A = 1 - \sin^2 A$ and substituting this in the above equation,

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta\{2(1-\sin^2\theta)-1\}}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

(viii) $\left(\sin A + \operatorname{cosec} A\right)^2 + \left(\cos A + \sec A\right)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S = $\left(\sin A + \operatorname{cosec} A\right)^2 + \left(\cos A + \sec A\right)^2$

By using $(a + b)^2 = a^2 + 2ab + b^2$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

By rearranging an using sec A = 1/cos A

$$= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$

Hence $(\sin^2 A + \cos^2 A)$ = 1 and $(\operatorname{cosec}^2 A + \sec^2 A)$ = 1

$$= (1) + \left(1 + \cot^2 A + 1 + \tan^2 A\right) + (2) + (2)$$
$$= 7 + \tan^2 A + \cot^2 A$$
$$= \text{R.H.S}$$

(ix) $\left(\operatorname{cosec} A - \sin A\right)\left(\sec A - \cos A\right) = \dfrac{1}{\tan A + \cot A}$

L.H.S = $\left(\operatorname{cosec} A - \sin A\right)\left(\sec A - \cos A\right)$------------Equation (1)

We know that the trigonometric functions,

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above values in Equation (1)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$
$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$
$$= \frac{\left(\cos^2 A\right)\left(\sin^2 A\right)}{\sin A \cos A}$$
$$= \sin A \cos A$$

R.H.S $= \dfrac{1}{\tan A \cot A}$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

By substituting the above function in RHS

$$= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}} = \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$
$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

By Identity $\sin^2 A + \cos^2 A = 1$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Hence, L.H.S = R.H.S

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2$

Taking LHS, $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{1}{\cos^2 A} \times \sin^2 A = \tan^2 A$$

Taking RHS, $\left(\frac{1-\tan A}{1-\cot A}\right)^2$

$$= \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$$

$$= (-\tan A)^2 = \tan^2 A$$

Hence, L.H.S = R.H.S.